



Test Fixture Effects in Vibration Tests of Rocket Motors

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Calspan Corporation, AEDC Division

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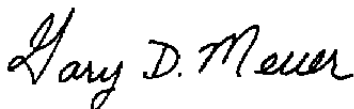
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PREFACE

The work reported herein was performed by the Arnold Engineering Development Center (AEDC), Air Force Systems Command (AFSC). The results were obtained by Calspan Corporation, AEDC Division, operating contractor for the aerospace flight dynamics testing effort at the AEDC, AFSC, Arnold Air Force Station, Tennessee under Project Number V44W83A. The Project Monitor was Lt. Gary Meuer. The research was performed from November 1, 1983 through May 1, 1983, and the manuscript was submitted for publication on April 24, 1985.

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1.0 INTRODUCTION

Vibration testing is an important part of any ground test program directed toward the validation of the flight readiness of a solid-propellant rocket motor. Such a motor may be utilized in either an aircraft-launched system or in a multistage booster system. In vibration testing of a rocket motor, the motor is exposed to a vibration environment simulating its anticipated flight environment prior to ignition. In this type of testing, a test fixture is required to permit adapting the rocket motor to an electrodynamic shaker which provides the required forcing function to the fixture-motor combination. The responses of the motor and its components to the ground test vibration environment are evaluated from measurements obtained using accelerometers mounted on the motor and on the test fixture. Anytime that an undesirable motor response is experienced during a test, possible adverse contributions of the fixture become a major concern. Such fixture concerns are related to fixture design problems which arise primarily because of the incompatibility of fixture weight and rigidity constraints. To minimize fixture effects in tests, a more rigid fixture is desired which corresponds, in general, to larger fixture weight. However, allowable fixture weight is normally restricted by the limited forcing capability of the shaker. It follows that required compromises in a fixture design can contribute to adverse characteristics of a test fixture which is normally provided by the ground test facility that is performing the vibration testing. The purpose of this report is to present the results of a study concerned with the interpretation of rocket motor response measurements and to define a means for identifying frequency ranges of potential adverse effects of test fixtures being used.

2.0 DISCUSSION

2.1 PROCEDURE IN VIBRATION TESTING

In vibration testing of a rocket motor, usually made over a frequency range of 5 to 2,000 Hz, the desired fixture acceleration level is maintained with use of a control system which monitors the outputs of control accelerometers (mounted on the test fixture in appropriate locations) and adjusts the input signal to the shaker consistent with the measured outputs of the control accelerometers. A photograph of a typical rocket motor, test fixture, and electrodynamic shaker system used in vibration testing in the horizontal plane is shown in Fig. 1. Although different modes of vibration testing are utilized, any adverse contributions of a test fixture can be best identified using the sine sweep mode in conjunction with a control system designed to maintain a constant fixture acceleration level over the frequency range. The typical frequency sweep rate is two octaves per minute.

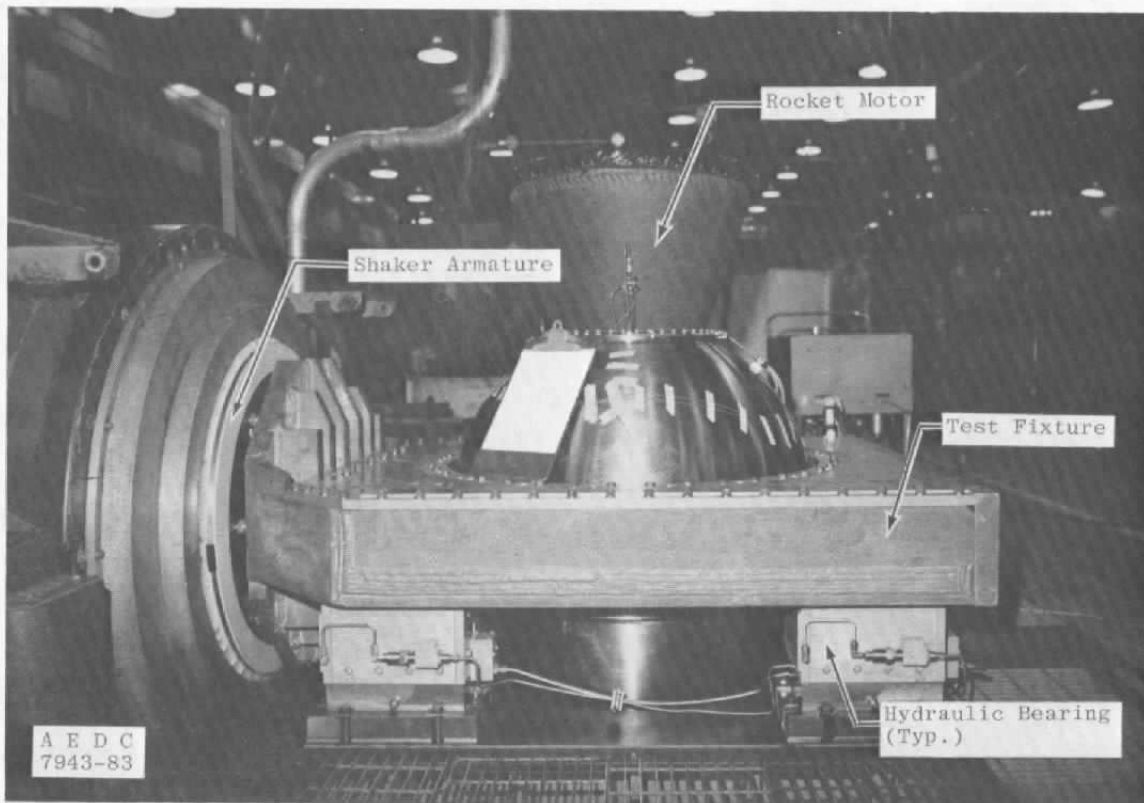


Figure 1. Typical electrodynamic shaker, test fixture, and rocket motor assembly.

2.2 IDENTIFICATION OF THE PROBLEM OF CONCERN

An acceleration response curve (sine sweep mode) for a rocket motor as a function of the forcing frequency that was obtained in a previous vibration test program, referred to here as the A-test program, is shown in Fig. 2. This curve is for a test performed in the horizontal plane and the corresponding input acceleration curve, the average of measurements from the control accelerometers positioned on the test fixture, is shown in Fig. 3. The large increase in the response acceleration for the motor in the region of 141 Hz caused appreciable concern. The cause for this undesirable response was attributed, at the time of the A-test program, to adverse test fixture effects. Test fixtures used in vibration tests in the horizontal plane are similar to the one shown in Fig. 1. The basic portion of such a fixture is of box-type construction and is mounted on four hydraulic bearings. Regions of high acceleration similar to that shown in Fig. 2 have been observed in tests of other rocket motors, and they normally occur within a frequency range of 100 to 200 Hz.

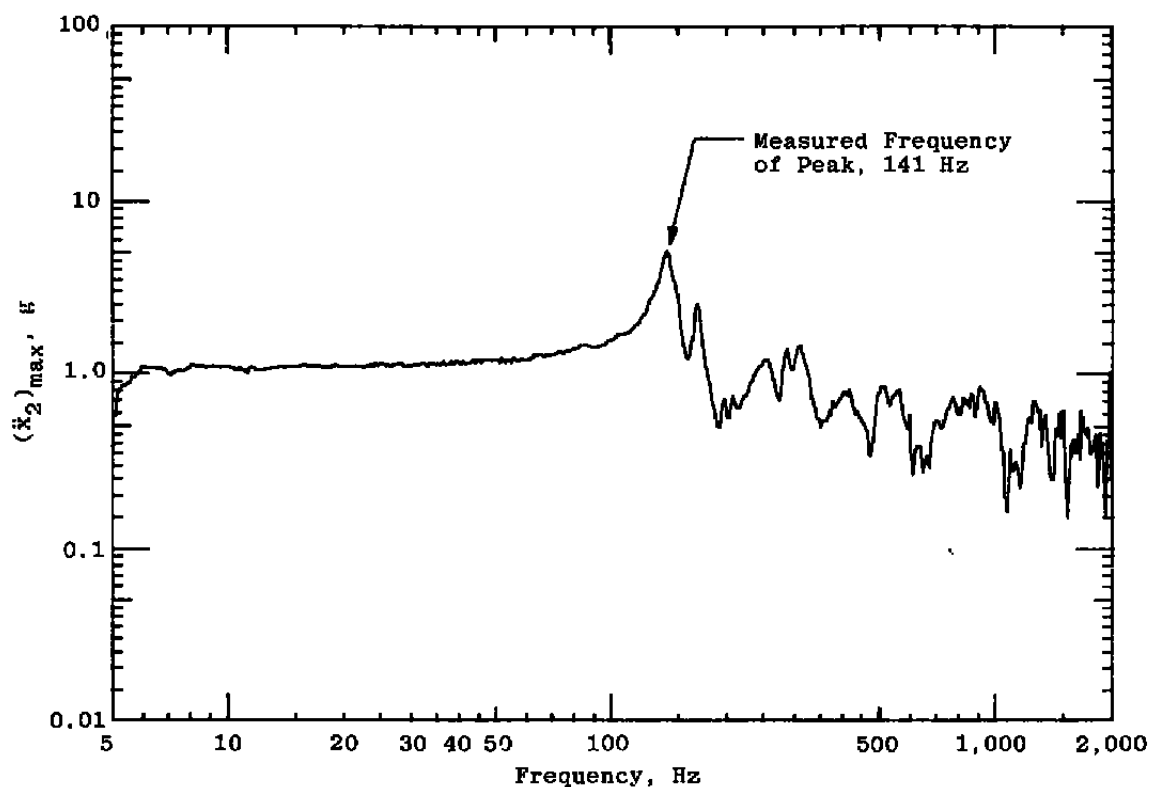


Figure 2. Representative acceleration response curve for a rocket motor—A-test program.

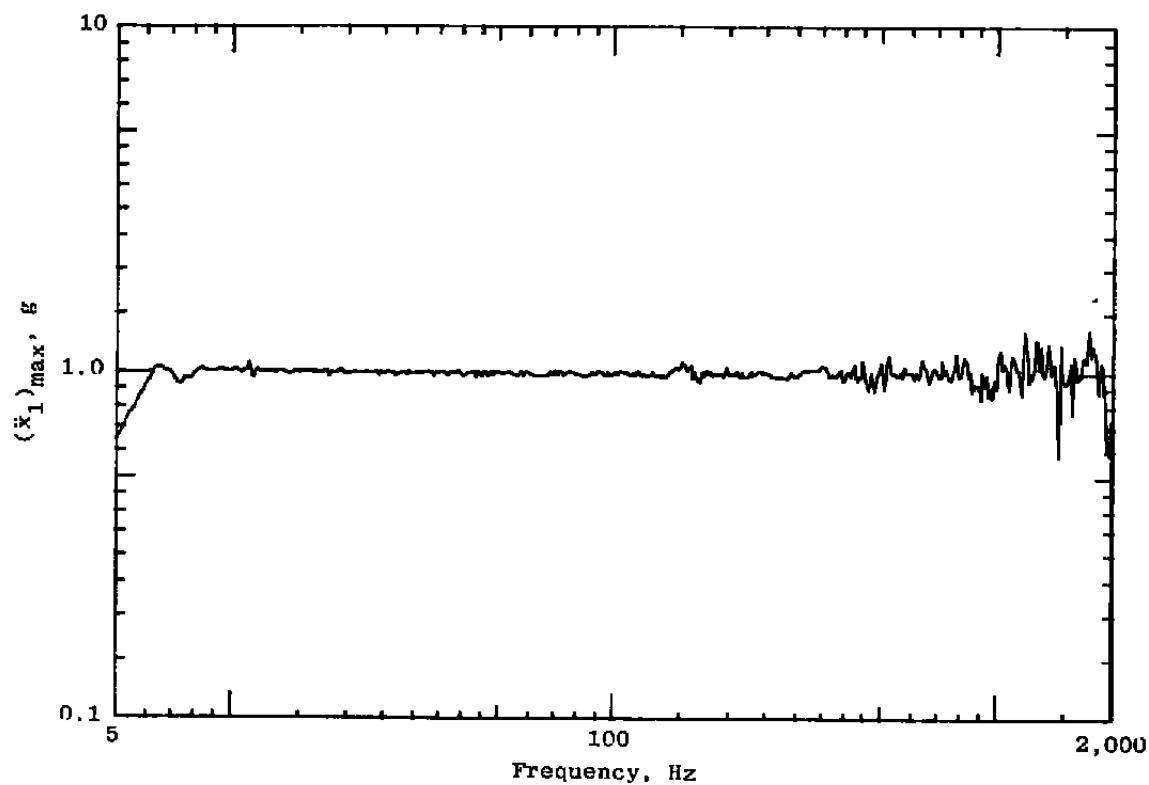


Figure 3. Acceleration curve for the test fixture—A-test program.

More recently, this problem area was examined at AEDC on the premise that the predominant motions of the shaker armature, test fixture, and rocket motor system can be adequately defined, below some frequency level, by the motion equations for a forced two-degree-of-freedom (DOF) system. Here, the combination of the test fixture and shaker armature having a high-strength bolted joint corresponds to the first mass (m_1) and the rocket motor corresponds to the second mass (m_2). The stiffness parameter (k_1) for m_1 is provided by the armature support flexures and the stiffness parameter (k_2) for m_2 is provided by the rocket motor mounting flange which attaches to the test fixture. Note that different types of motor mounting flanges are utilized. In some flange designs, the k_2 stiffness parameter is completely provided by the flange material, whereas in other designs, k_2 can also be dependent on the array of bolts used in attaching the flange to the test fixture. The results of the examination are discussed herein.

2.3 MOTION EQUATIONS FOR A TWO-DOF SYSTEM

To permit clarification of the basic characteristics of a two-DOF vibration system, the motion equations for such a system are listed and discussed. Consistent with the sketches of Fig. 4 showing a simplified rocket motor vibration system and the corresponding free-body forces involved in a forced, zero-damped, two-DOF system, the equations of motion for the first mass, m_1 , and for the second mass, m_2 , can be written

$$\begin{aligned} m_1 \ddot{x}_1 &= -k_1 x_1 + k_2 (x_2 - x_1) + F \\ m_2 \ddot{x}_2 &= -k_2 (x_2 - x_1) \end{aligned}$$

and rearranging terms

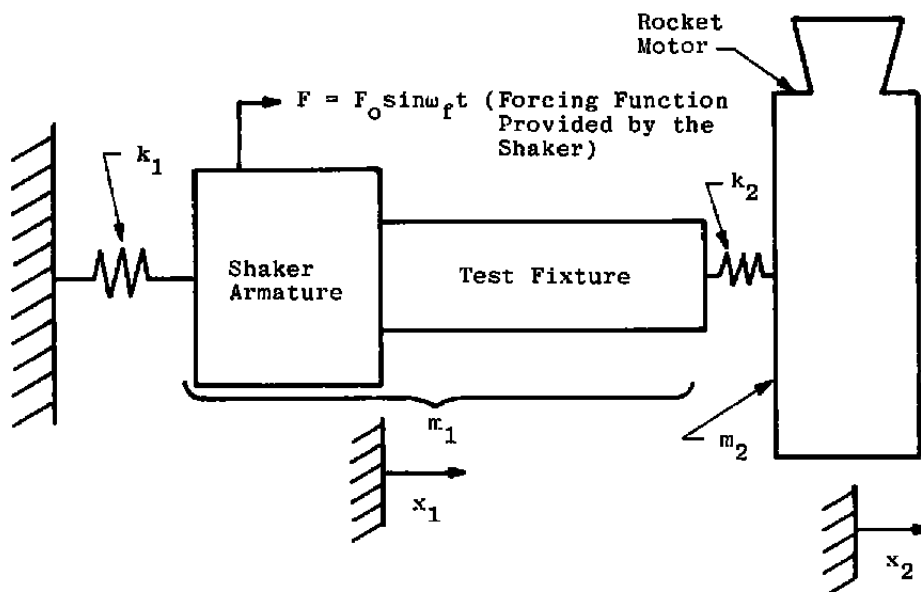
$$m_1 \ddot{x}_1 + (k_1 + k_2)x_1 - k_2 x_2 = F \quad (1)$$

$$m_2 \ddot{x}_2 - k_2 x_1 + k_2 x_2 = 0 \quad (2)$$

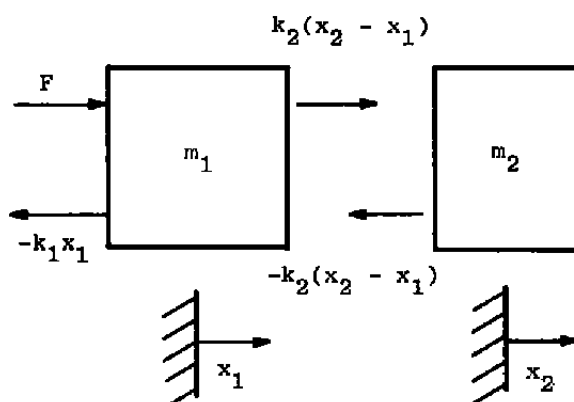
here, \ddot{x}_1 and \ddot{x}_2 are accelerations of m_1 and m_2 , respectively. The particular solutions for these equations, using $F_0 \sin \omega t$ for the forcing function, F , can be written

$$x_1 = X_1 \sin \omega t$$

$$x_2 = X_2 \sin \omega t$$



a. Two-DOF rocket motor vibration system



- Notes: k_1 - Stiffness Parameter Provided by the Armature Support Flexures
 k_2 - Stiffness Parameter Between the Rocket Motor and the Test Fixture Resulting from the Motor Attachment Flange
 m_1 - Mass of the Shaker-Armature and the Test Fixture Combination Which Utilizes a High-Strength Bolted Joint
 m_2 - Mass of the Rocket Motor
 x_1, x_2 - Displacements of m_1 and m_2 , respectively

b. Corresponding free-body diagram

Figure 4. Simplified rocket motor vibration system and corresponding forces involved.

substituting these expressions for x_1 and x_2 into Eqs. (1) and (2),

$$-m_1\omega_f^2 X_1 + (k_1 + k_2) X_1 - k_2 X_2 = F_0$$

$$-m_2\omega_f^2 X_2 - k_2 X_1 + k_2 X_2 = 0$$

and rearranging terms provide the following equations:

$$(-m_1\omega_f^2 + k_1 + k_2) X_1 - k_2 X_2 = F_0 \quad (3)$$

$$(-m_2\omega_f^2 + k_2) X_2 - k_2 X_1 = 0 \quad (4)$$

Incorporating the substitutions

$\omega_1 = \sqrt{k_1/m_1}$, natural frequency of the decoupled m_1 mass ($k_2 = 0$)

$\omega_2 = \sqrt{k_2/m_2}$, natural frequency of the decoupled m_2 mass (m_1 fixed)

$X_0 = F_0/k_1$, static deflection of m_1 for a static force F_0 .

into Eqs. (3) and (4), expressions for the maximum displacements, X_1 and X_2 of m_1 and m_2 , respectively, can be written

$$X_1 = \frac{(1 - \omega_f^2/\omega_2^2)X_0}{[(1 + k_2/k_1 - \omega_f^2/\omega_1^2)(1 - \omega_f^2/\omega_2^2) - k_2/k_1]} \quad (5)$$

$$X_2 = \frac{X_0}{[(1 + k_2/k_1 - \omega_f^2/\omega_1^2)(1 - \omega_f^2/\omega_2^2) - k_2/k_1]} \quad (6)$$

or

$$X_2 = X_1/(1 - \omega_f^2/\omega_2^2) \quad (7)$$

Note in Eq. (5) that when the forcing frequency, ω_f , is equal to ω_2 that X_1 tends to go to zero, and from Eq. (6) that

$$X_2 = X_0/(-k_2/k_1) = -F_0/k_2 \quad (7a)$$

These are very significant observations in conjunction with the sine sweep testing mode as again the control system is designed to keep the maximum acceleration of the first mass, $(\ddot{x}_1)_{\max}$, at a predefined constant level over the frequency range. Note for a given frequency

that $(\ddot{x}_1)_{\max}$ and $(\ddot{x}_2)_{\max}$ are directly proportional to X_1 and X_2 , respectively. Hence, as ω_f is swept through ω_2 , the control system adjusts the shaker input signal to increase F_0 to a peak value at $\omega_f = \omega_2$ in attempting to keep $(\ddot{x}_1)_{\max}$ at its predefined level. At this same testing condition, ($\omega_f = \omega_2$), the acceleration of the second mass, $(\ddot{x}_2)_{\max}$ will increase to a peak value consistent with the peak value for X_2 as defined by Eq. (7a).

Also observe in Eqs. (5) and (6) that for $\omega_f \neq \omega_2$, that both X_1 and X_2 will tend to become infinite when the denominator in the expressions for X_1 and X_2 is zero. The corresponding forcing frequency relationship for this condition can be written

$$\omega_f/\omega_2 = \sqrt{\frac{1}{2}[(\omega_1^2/\omega_2^2) + (m_2/m_1) + 1]} \pm \sqrt{\left\{\frac{1}{2}[(\omega_1^2/\omega_2^2) + (m_2/m_1) + 1]\right\}^2 - \omega_1^2/\omega_2^2} \quad (8)$$

The two values of ω_f defined by this expression are the natural frequencies of the two-DOF system; the larger value of ω_f to satisfy this condition is designated ω_2' in this report, and the smaller value is designated ω_1' . It follows that for the forcing frequency at either ω_1' or ω_2' (again both X_1 and X_2 are experiencing resonant conditions at these frequencies) that the shaker force, F_0 , required will necessarily be at a local minimum level.

The more important observations from the above discussion are that the motion equations for a forced two-DOF system with a control system designed to maintain the first mass at a constant acceleration level over the frequency range indicate (1) at $\omega_f = \omega_2$, there will be detectable peaks in both the $(\ddot{x}_2)_{\max}$ and F_0 curves, and (2) there will be detectable local minimums in the F_0 curve at $\omega_f = \omega_1'$ and at $\omega_f = \omega_2'$.

2.4 EXPERIMENTAL DATA FOR A TWO-DOF SYSTEM

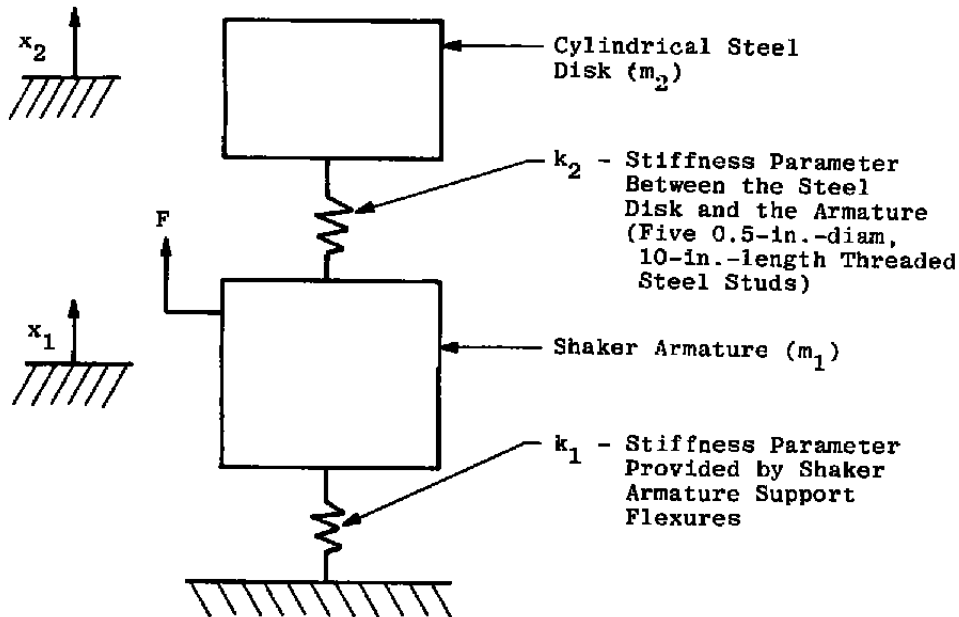
To demonstrate the usefulness of the motion equations in examining experimental two-DOF systems, a limited amount of experimental data using the sine mode of testing was obtained in an experiment for the near-ideal two-DOF system defined in Fig. 5. Measured input signals for the system (P_0 and P_0') are shown in Fig. 6. Using the listed values in Fig. 5 for m_1 , m_2 , k_1 , and k_2 , the following calculated values for the natural frequency of the decoupled second mass, ω_2 , and the natural frequencies ω_1' and ω_2' of the two-DOF system were obtained, using Eq. (8),

$$\omega_2 = 137.8 \text{ Hz}$$

$$\omega_1' = 8.7 \text{ Hz}$$

$$\omega_2' = 271.1 \text{ Hz}$$

In Fig. 7, the measured acceleration curve of the second mass, $\ddot{x}_{2\max}$, is shown as a function of frequency for a testing condition for which the control system was preset to provide an input acceleration to the first mass, (m_1) of 0.5 g. The corresponding measured input signal



Notes: m_1 - 12.42 Slugs, Manufacturer Listed Value

m_2 - 35.65 Slugs, Measured

k_1 - 1.44×10^5 lb/ft, Manufacturer Listed Value

k_2 - 2.62×10^7 lb/ft, Calculated Using a Modulus of Elasticity of 3.0×10^7 for Steel Along with the Minimum Diameter and Nominal Length of the Threaded Studs

$F = F_0 \sin \omega_f t$ (Forcing Function)

Figure 5. Two-degree-of-freedom vibration system experiment.

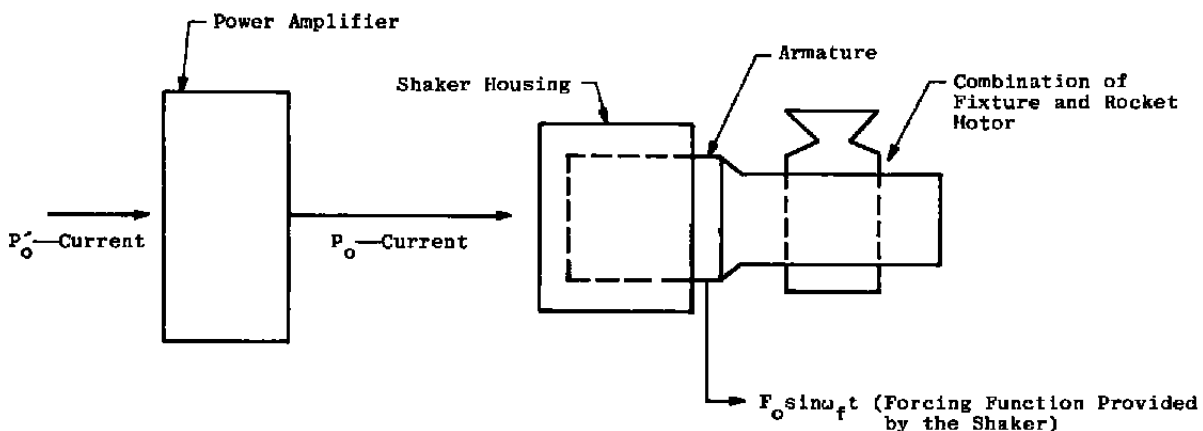


Figure 6. P_0 and P_0' measurements.

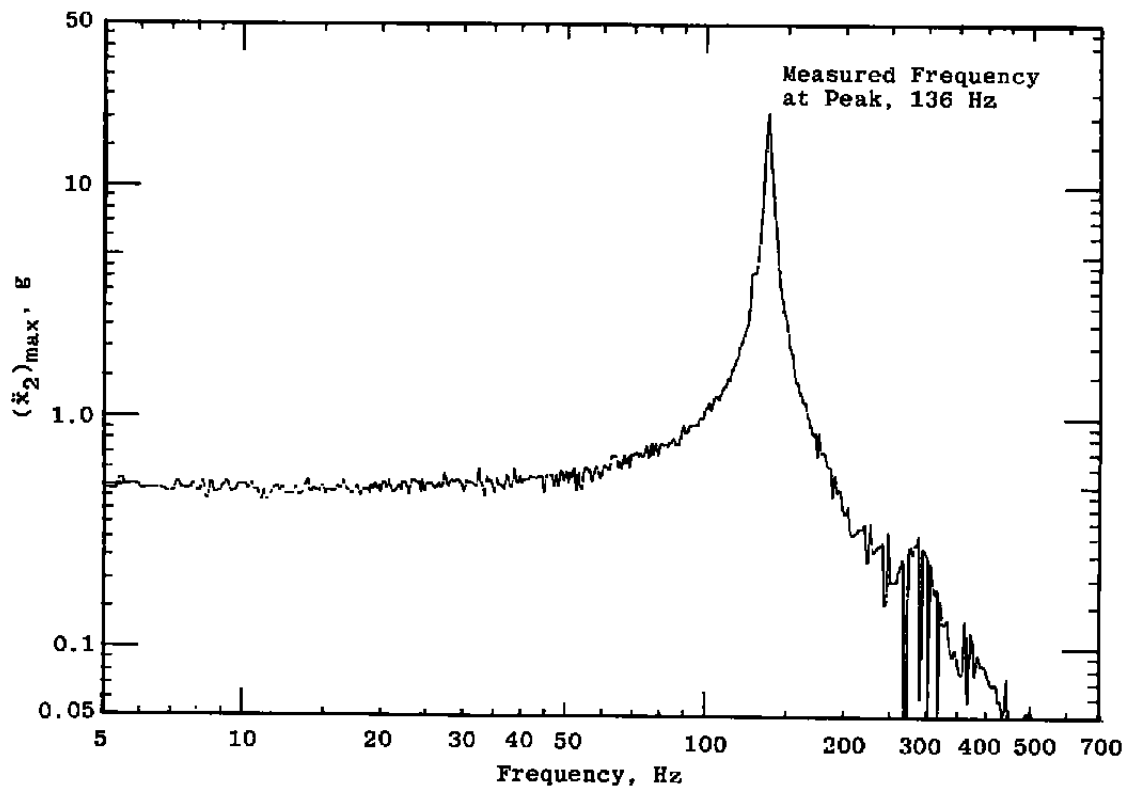
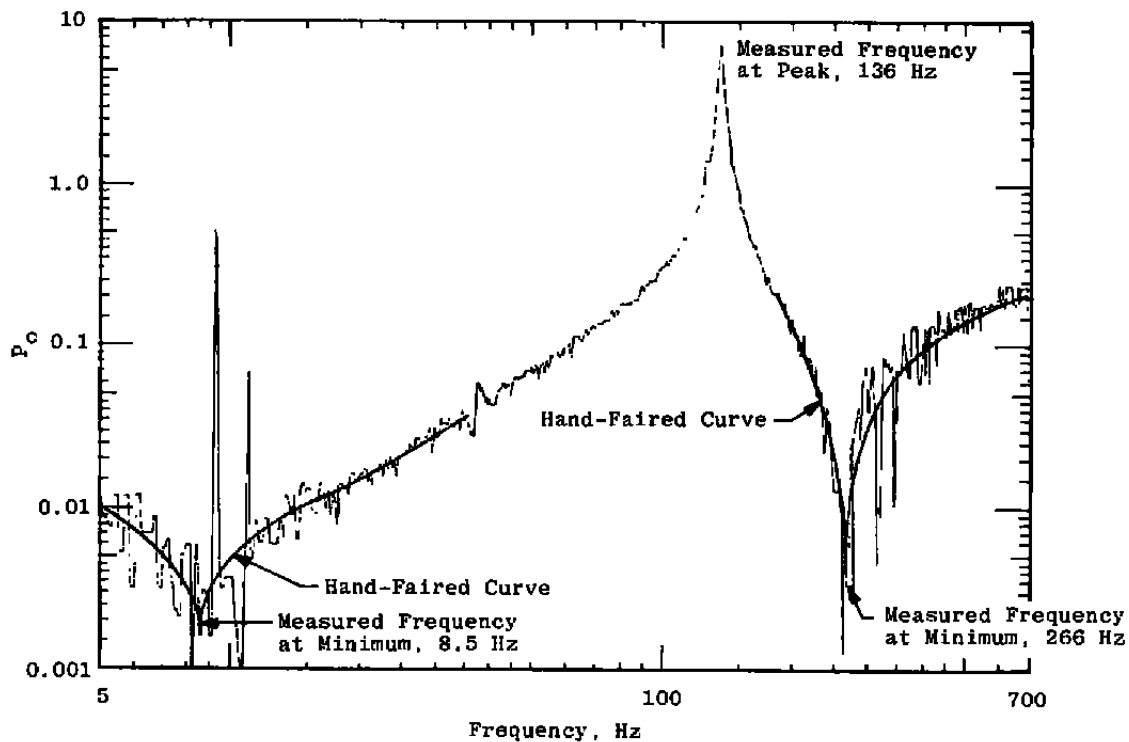


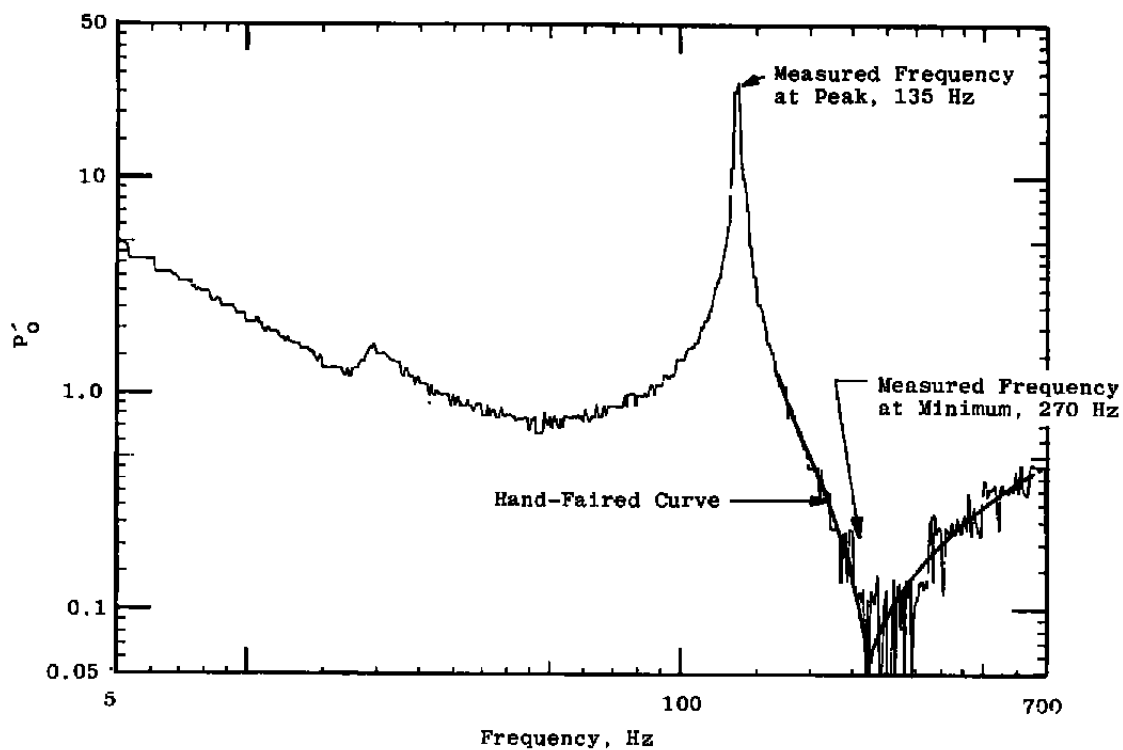
Figure 7. Acceleration response curve for the second mass of the two-degree-of-freedom system experiment.

to the shaker, P_o , which is directly proportional to F_o (local maximum force applied by the shaker to the first mass) is shown in Fig. 8a. It is apparent that a very significant peak in $(\ddot{x}_2)_{\max}$ occurs at the measured frequency of 136 Hz, and the corresponding shaker input signal, P_o , also has a well-defined peak at the measured frequency of 136 Hz. Note that the two measured frequencies corresponding to the peaks in the $(\ddot{x}_2)_{\max}$ and P_o curves agree with the calculated frequency for ω_2 of the experimental system as would be expected. It follows that the measurements for the near-ideal two-DOF experimental system are consistent with the corresponding motion equations, and that at $\omega_f = \omega_2$ (the natural frequency of the decoupled second mass), the natural behavior of such a system is to cause a significant increase in $(\ddot{x}_2)_{\max}$ and a corresponding significant increase in P_o . Further, observe in Fig. 8a that the P_o curve has two well-defined local minimums at ω_f values of 8.5 and 266 Hz. As expected, these frequencies agree with the calculated natural frequencies of the two-DOF system, ω'_1 and ω'_2 , listed above. It should be noted that the capability in computing ω_2 , ω'_1 , and ω'_2 , for the experimental system is restricted by the limitation in defining the value of k_2 for the threaded studs used.

The corresponding P'_o parameter is shown in Fig. 8b and is related to the input signal to the shaker, P_o ; again P_o is the measured input signal to the shaker, whereas P'_o is the measured input signal to the corresponding shaker power amplifier. From a comparison of



a. Shaker input signal, P_o



b. Power amplifier input signal, P'_o

Figure 8. Shaker inputs for the two-degree-of-freedom system experiment.

figures, 8a and 8b, it is apparent that the P'_0 curve is quite adequate at frequencies above about 100 Hz for defining the basic trends associated with the P_0 curve, whereas at the lower frequencies the P'_0 measurements are invalid as they include significant power amplifier effects. The consistencies in the P'_0 and P_0 variations above 100 Hz are noted because in previous test programs, only P'_0 measurements were available; in more recent tests, P_0 measurements have been obtained as a standard procedure.

2.5 ROCKET MOTOR MEASUREMENTS

It has been well recognized that rocket motor test fixtures can behave adversely above some frequency level because of resonant characteristics inherent in such structures. Hence, it is of much importance to be able to define the frequency level above which such effects are nontrivial. In Fig. 9, the input acceleration $(\ddot{x})_{\max}$ for a typical fixture-armature combination, is shown as a function of frequency. This curve indicates for frequencies up to about 500 Hz, except for a couple of negative spikes at frequencies of about 280 to 350 Hz, that the test fixture behaves quite well. However, in Fig. 10a the corresponding P_0 curve is shown which indicates that the control system required appreciable excursions in the shaker input signal, starting at about 250 Hz. Such required excursions in P_0 , though $(\ddot{x})_{\max}$ was controlled reasonably well, indicate a tendency of the fixture for adverse behavior, and hence, the use

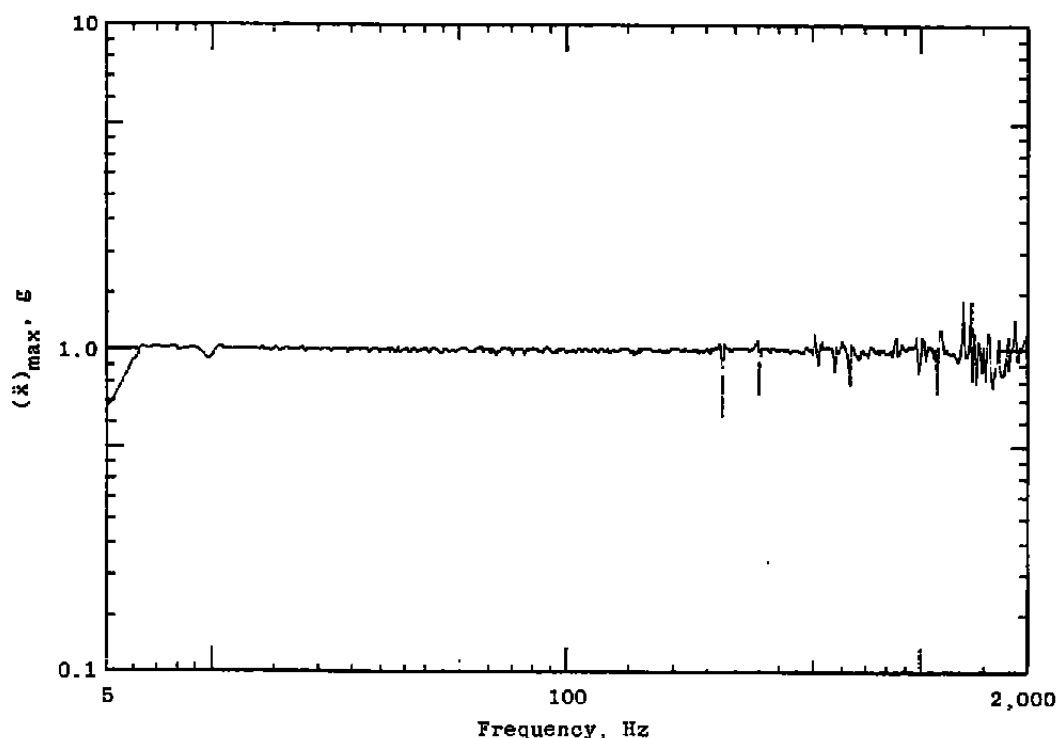
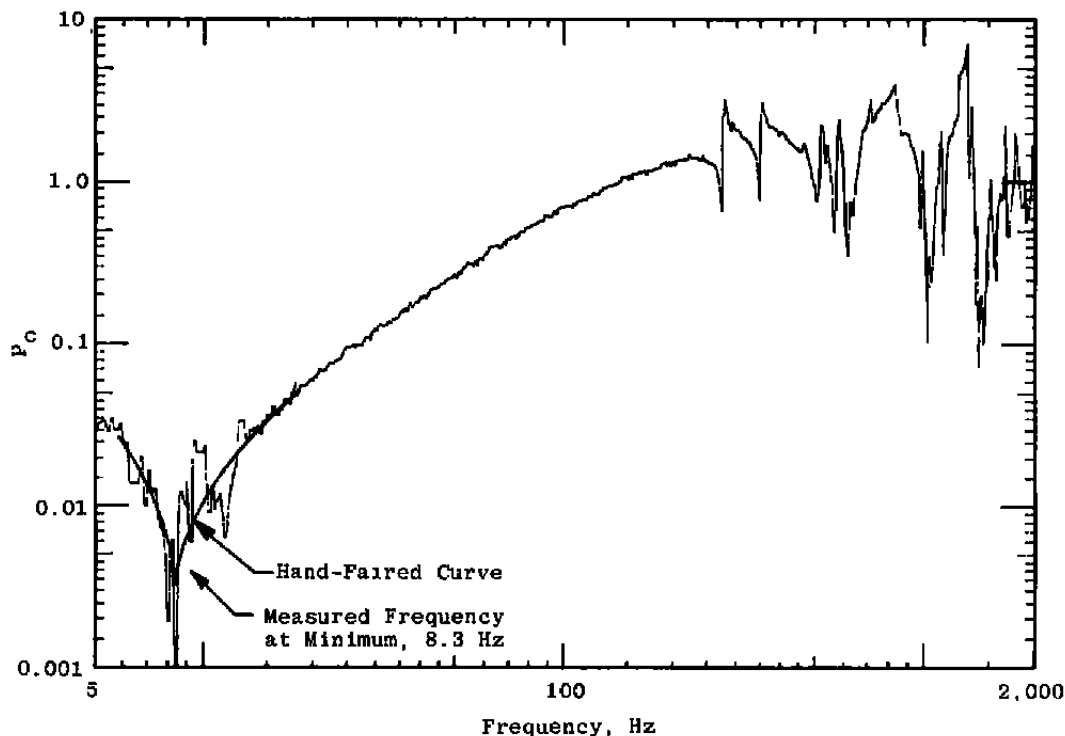


Figure 9. Acceleration curve for a typical armature-fixture combination.

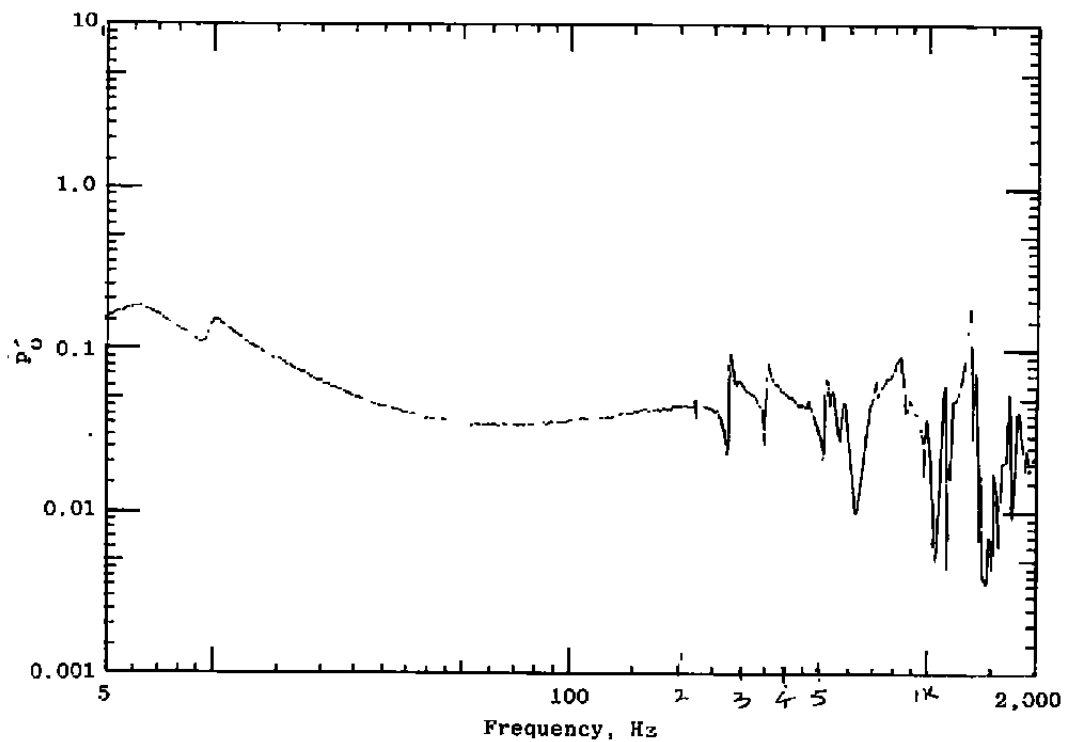
of the test fixture above 250 Hz could be questionable. It follows that P_o is a sensitive parameter to test fixture behavior and can be expected to provide a reasonably good means for detection of the initiation of any adverse test fixture behavior. The corresponding P_o' curve is shown in Fig. 10b to provide further evidence of its agreement with P_o measurements at the higher frequencies. The P_o' curve for the armature-fixture combination that was used in the A-test program (see Fig. 2) is presented in Fig. 11a. This curve indicates that the test fixture used in that test program had no significant adverse motion below a frequency of about 200 Hz.

The measured P_o' curve corresponding to the rocket motor $(\ddot{x}_2)_{\max}$ curve of Fig. 2 is shown in Fig. 11b; P_o measurements were not available in the A-test program. Note that the peak in the P_o' curve measured at a frequency of 141 Hz is consistent with the peak in the $(\ddot{x}_2)_{\max}$ curve at 141 Hz in Fig. 2. These peaks observed in the $(\ddot{x}_2)_{\max}$ and P_o' curves of Figs. 2 and 11b, in consideration of the discussion of the motion equations of Section 2.3 and the experimental data of Section 2.4, indicate that the rocket motor vibration system (Fig. 2) behaved predominantly as a two-DOF system at least up through a forcing frequency corresponding to the peaks in the $(\ddot{x}_2)_{\max}$ and P_o' curves. This is a very important observation as the forcing frequency corresponding to the peaks in the $(\ddot{x}_2)_{\max}$ and P_o' curves is ω_2 which,



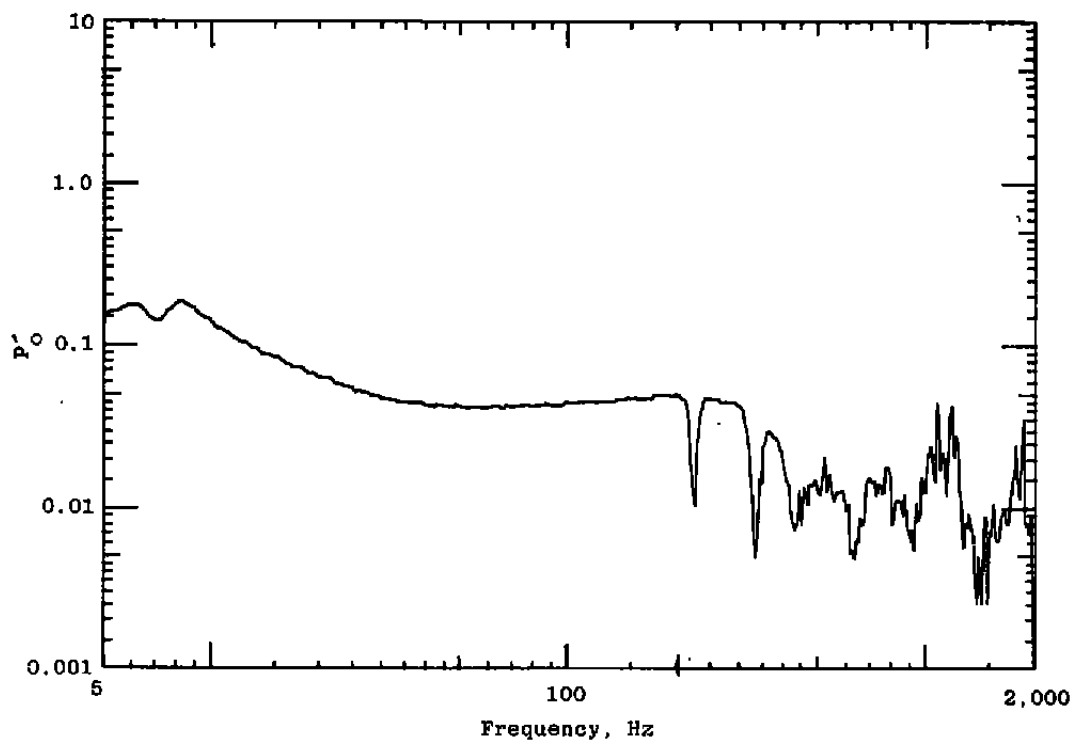
a. P_o

Figure 10. Shaker inputs for armature-fixture test of Fig. 9.



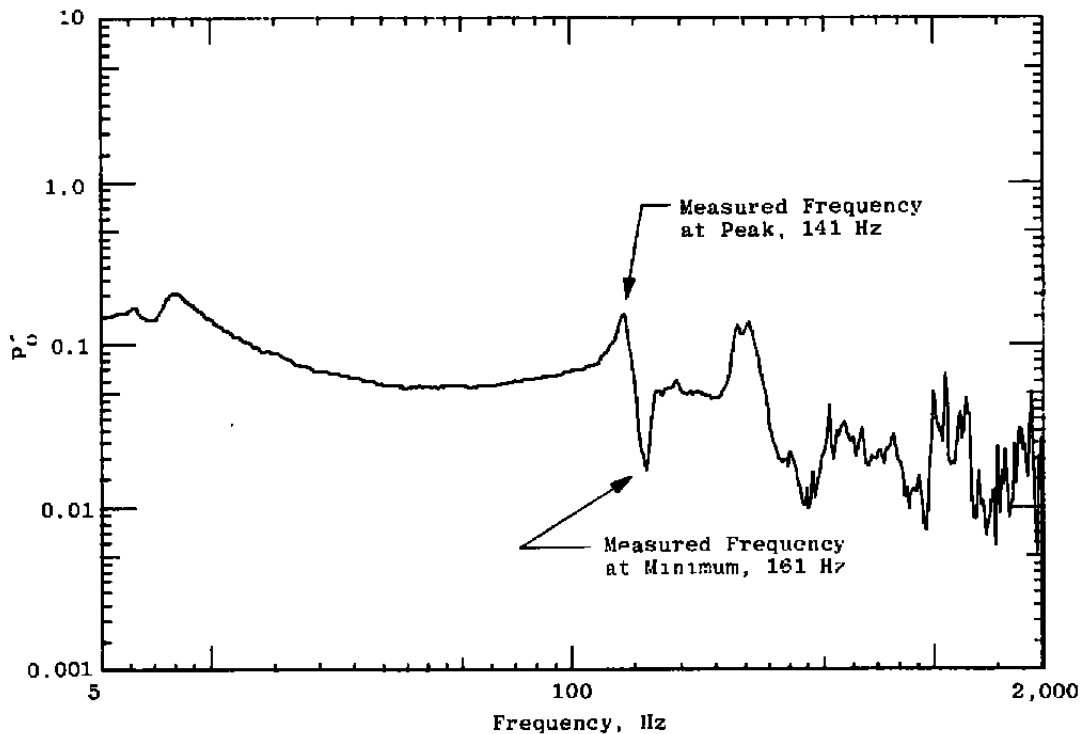
b. P'_0

Figure 10. Concluded.



a. Armature-fixture combination

Figure 11. Inputs corresponding to the acceleration response curve of Fig. 2.



b. Complete rocket motor system

Figure 11. Concluded.

again, is the natural frequency of the uncoupled second mass (rocket motor). As discussed previously, an increase in $(\dot{x}_2)_{\max}$ at $\omega_f = \omega_2$ is a basic characteristic of a forced two-DOF system.

Considering the current analysis, the increase in $(\dot{x}_2)_{\max}$ observed for the rocket motor in Fig. 2 (A-test program) would be expected. It follows then that the large increase in the acceleration response of the rocket motor at $\omega_f \approx 141$ Hz (Fig. 2) should not be attributed to adverse test fixture effects.

Although there is a well-defined local minimum in the P'_0 curve of Fig. 11b, consistent with the motion equations of Section 2.3, at a frequency that agrees well with a calculated ω'_2 (using an ω_2 value defined by peaks in the $(\dot{x}_2)_{\max}$ and P'_0 curves), this will not be the case, in general, in rocket motor vibration tests. Usually, ω'_2 will be from 10 to 50 percent larger than ω_2 and, hence, nearer to a frequency level involving nontrivial fixture effects. A testing condition having possible small fixture effects combined with the control system operating at a low-level force requirement (resonant condition) can produce a meaningless variation in the shaker input in the ω'_2 frequency range. It should also be noted that the apparent large noise level in some of the shaker input measurements is related to the low signal levels involved. Further, damping effects ignored in the present analysis would not be expected to have any significant effects on the results of the analysis.

3.0 CONCLUDING REMARKS

Results of an analysis of rocket motor acceleration response measurements from vibration tests indicate that large, undesirable motor accelerations may be measured and invalidly attributed to the test fixture used. Further, the use of the measured input signal to the shaker in forced vibration tests of rocket motors (not normally measured in the past) can be very useful in identifying the forcing frequency level above which significant adverse effects of a given test fixture can be expected.

NOMENCLATURE

| | |
|--------------------------|---|
| F | Forcing function, $F_0 \sin \omega_f t$ |
| F_0 | F_{\max} |
| k_1 | Stiffness parameter of the m_1 mass |
| k_2 | Stiffness parameter of the m_2 mass |
| m_1 | First mass in a two-degree-of-freedom system |
| m_2 | Second mass in a two-degree-of-freedom system |
| P_0 | Shaker input current |
| P'_0 | Power amplifier input current |
| X_0 | Static displacement of the m_1 mass, (F_0/k_1) |
| X_1 | $(x_1)_{\max}$ |
| X_2 | $(x_2)_{\max}$ |
| x_1 | Displacement of the m_1 mass |
| x_2 | Displacement of the m_2 mass |
| \ddot{x}_1, \ddot{x}_2 | Accelerations in the x_1 and x_2 directions, respectively |
| ω_f | Forcing frequency |

| | |
|-------------|--|
| ω_1 | Natural frequency of the decoupled m_1 mass |
| ω_2 | Natural frequency of the decoupled m_2 mass |
| ω_1' | First natural frequency of a two-degree-of-freedom system |
| ω_2' | Second natural frequency of a two-degree-of-freedom system |